Book Review

Reiner Horst and Hoang Tuy, *Global Optimization – Deterministic Approaches*, 2nd Edition, Springer-Verlag, Berlin, 1993, 698 pages.

An enormous number of practical problems encountered in such diverse areas as engineering, business, economics and physics can be represented as constrained global optimization problems. This has motivated the development of a large and increasing number of algorithms for constrained global optimization. Coupled with a rapidly-improving computer technology, these algorithms have been used to solve increasingly complex problems and applications. These successes have, in turn, motivated practioners and theoreticians to continually develop new global optimization applications and algorithms.

This book provides an insightful and unified presentation of deterministic methods of constrained global optimization. It focuses on those methods that exploit certain helpful problem structures that may be present. Before presenting the methods, the book provides a coherent view of the fundamental approaches that serve to unite them. This significantly facilitates the individual explanations of each of the algorithms that follow.

The majority of the algorithms are from the open literature. However, variations on these algorithms and some new algorithms are occasionally given. For instance, the Concave Polyhedral Underestimation algorithm for concave minimization over a polytope and the branch and bound algorithm for reverse convex programs with d.c. objective functions are new algorithms.

Computational strengths and weaknesses of most of the algorithms are discussed. These discussions are useful in their own right. However, they are often also used to motivate and compare algorithms designed for the same problem class. In some cases, actual computational results from the literature are provided. However, the authors warn that these should be considered preliminary.

The presentation is accessible to readers in a variety of disciplines since the technical prerequisites are quite minimal. They include a sound knowledge of elementary real analysis, linear algebra, and convexity theory. No knowledge of any other branch of mathematics is required.

The book is divided into three major parts. Each part consists of from two to five chapters.

Part A first defines the four general subclasses of constrained global optimization problems of primary concern in the book. These are concave minimization, reverse convex programming, d.c. programming, and Lipschitzian optimization. Basic properties and some applications of each subclass are given. Emphasis is placed on those properties that make these subclasses more amenable to global

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solution than general global optimization problems. Part A also shows how d.c. programming and Lipschitzian optimization methods can be adapted to solving systems of nonlinear equations and inequalities.

Next, part A provides thorough presentations and discussions of each of three fundamental techniques often used in global optimization: outer approximation, concavity cuts and branch and bound. Each technique is first presented in full generality, after which particularizations and specific classes of implementations are discussed. These three basic techniques serve to unite and facilitate the presentations of the algorithms to follow.

Part B is devoted to the presentation of concave minimization algorithms. Most of the popular algorithms for concave minimization are described and analyzed either directly or indirectly in this part. The presentation is organized according to the main approach used by each algorithm. The approaches considered employ cuts, successive approximation and successive partitioning (branch and bound). Part B also describes several decomposition algorithms for large scale concave minimization problems and various algorithms for certain specially-structured problems that can be converted to and solved as concave minimization problems. The latter problems include bilinear programs, linear and concave complementarity problems and parametric concave minimization problems (i.e., linear reverse convex programs).

Part C provides descriptions of a number of algorithms for solving d.c. programs, Lipschitzian optimization problems and the more general global optimization problem of minimizing a continuous function over a compact set. Several outer approximation algorithms, branch and bound procedures and combinations thereof are presented for solving d.c. programs and Lipschitzian optimization problems. The Relief Indicator Method for minimizing continuous functions over compact sets is also described.

This is a beautifully written book by two acknowledged leaders in the field of global optimization. The concepts underlying the various techniques and approaches are presented in a lucid and elegant manner. Motivational discussions are coherently written, and the theoretical and algorithmic presentations are carefully crafted in an accurate and complete fashion. The unifying discussions of the fundamental techniques given in Part A are profoundly helpful in facilitating the ease with which the reader comprehends the essence and details of the algorithms described in Parts B and C. Furthermore, the coverage of the literature on the theory, solution and applications of global optimization is remarkably thorough.

The book is a joy to read and will undoubtedly become one of the most important standard research sources in global optimization. Since its mathematical prerequisites are modest, teachers and students of the field will also find it to be of use in a variety of graduate and advanced undergraduate programs. For instance, I have had great success using the book as the major source in my doctoral business course in global optimization. I highly recommend this out-

standing book to anyone interested in deterministic algorithms for constrained global optimization problems.

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